On the Relation Between Remainder Set and Kernel

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Outline of Topics

Belief Revision

- Updating a Knowledge Base
  - Expansion: Add new piece of knowledge
  - Contraction: Remove a piece of knowledge
  - Revision: Add a new piece of knowledge in a consistent way

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  - Expansion: Add new piece of knowledge
  - Contraction: Remove a piece of knowledge
  - Revision: Add a new piece of knowledge in a consistent way
- Separate the construction from the postulates
  - Representation Theorem

AGM Paradigm

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Partial Meet Contraction

Definition (Remainder Set)

$B' \in B \perp \alpha$ iff:

- $B' \subseteq B$
- $\alpha \notin Cn(B')$
- If $B' \subset B'' \subseteq B$ then $\alpha \in Cn(B'')$

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Definition (Selection Function)

A function γ is a selection function if it satisfies:

- $\emptyset \neq \gamma(B \perp \alpha) \subseteq B \perp \alpha$ if $B \perp \alpha \neq \emptyset$
- $\gamma(B \perp \alpha) = \{B\}$ otherwise

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Definition (Partial Meet Contraction)

$B -_\gamma \alpha = \bigcap \gamma(B \perp \alpha)$

Postulates for Partial Meet Contraction

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- **(success)** $\alpha \notin Cn(B - \alpha)$
- **(inclusion)** $B - \alpha \subseteq B$
- **(relevance)** If $\beta \in B \setminus B - \alpha$ then there is B' such that $B - \alpha \subseteq B' \subseteq B$ and $\alpha \notin Cn(B')$, but $\alpha \in Cn(B' \cup \beta)$
- **(uniformity)** If for all subsets B' of B it holds that $\alpha \in Cn(B')$ iff $\beta \in Cn(B')$ then $B - \alpha = B - \beta$.

Kernel Contraction

Definition (Kernel)

$B' \in B \perp\!\!\!\perp \alpha$ iff:

- $B' \subseteq B$
- $\alpha \in Cn(B')$
- If $B'' \subset B'$ then $\alpha \notin B''$

Kernel Contraction

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Definition (Incision Function)

A σ is an incision function if it satisfies:

- $\sigma(B \perp\!\!\!\perp \alpha) \subseteq \bigcup(B \perp\!\!\!\perp \alpha)$
- If $\emptyset \neq X \in B \perp\!\!\!\perp \alpha$ then $X \cap \sigma(B \perp\!\!\!\perp \alpha) \neq \emptyset$

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Definition (Kernel Contraction)

$B -_\sigma \alpha = B \setminus \sigma(B \perp\!\!\!\perp \alpha)$

Postulates for Kernel Contraction

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- **(success)** $\alpha \notin Cn(B - \alpha)$
- **(inclusion)** $B - \alpha \subseteq B$
- **(core-retainment)** If $\beta \in B \setminus B - \alpha$ then there is B' such that $B' \subseteq B$ and $\alpha \notin Cn(B')$, but $\alpha \in Cn(B' \cup \beta)$
- **(uniformity)** If for all subsets B' of B it holds that $\alpha \in Cn(B')$ iff $\beta \in Cn(B')$ then $B - \alpha = B - \beta$.

Minimal Cuts

Definition (Cut)

A **cut** in a class of sets B is a set B' that contains at least one element of each set

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Theorem

$\beta \in B \perp \alpha$ iff there is a minimal cut β' of $B \perp\!\!\!\perp \alpha$ and $\beta = B \setminus \beta'$

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- This result sugests that the kernel have at least the same amount of information as the remainder set (maybe more)
- Finding minimal cuts can be done with the well known Reiter's algorithm

Future Work

- Empirical tests:
  - Is it better to find the remainder set from the kernel or find each of them separatly?

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  - Is it faster to find the kernel or the remainder set?
- Theoric work:
  - Is it possible to find the kernel from the remainder set without using the theorem prover?