Base Revision in Description Logics - Preliminary Results

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Abstract. Belief Revision deals with the problem of adding new information to a knowledge base in a consistent way. The theory has been developed having in mind classical logics. In this paper, we show some problems of applying belief revision methods directly to ontologies represented in description logics and propose new operations that overcome these problems.

1 Introduction

We have seen in recent years more and more attention devoted to the issue of representing a domain of application by means of ontologies. After a period when the main interest was on building new ontologies, lately there have been some efforts towards tools for changing, repairing and maintaining ontologies.

Since knowledge is not static, there is a necessity to deal with the evolution of ontologies. When ontologies evolve, inconsistency may arise. There are many approaches concerning how to deal with inconsistencies. In [1], a unifying framework was described that accommodates four different approaches:

- *Consistent evolution* prevents the introduction of inconsistencies in a consistent ontology.
- *Repairing* makes an inconsistent ontology consistent.
- *Reasoning with inconsistency* tries to derive meaningful conclusions from an inconsistent ontology.
- *Versioning* keeps track of changes and compatibility issues between different versions of the ontology.

Belief revision theory [2, 3] addresses the first two approaches for dealing with inconsistency: preventing the introduction of inconsistency in a knowledge base and repairing an inconsistent knowledge base. In addition to studying constructions for operations of revision in knowledge bases, belief revision studies postulates that this constructions must satisfy. There are some proposals for constructing revision operators for ontologies, however almost none which takes care of the formal properties that the operations satisfy. For this reason it became important to study how to apply belief revision techniques in ontologies.

Alchourrón, Gärdenfors and Makinson in [4] proposed a set of postulates that every operation of belief revision should satisfy. This set of postulates, together with some proposed constructions, is know as AGM paradigm due to the authors' initials. Although this is the most influential work in the area, when we try to apply it to ontologies we have some problems: first the beliefs of an agent are represented by belief sets, sets of sentences closed under the consequence operator. Dealing with closed sets is a problem because they are very often infinite. The second problem was presented in [5]. The author showed in this work that some description logics can not satisfy the AGM postulates, they are not AGM-compliant. In particular SHOIN and SHIFF, the description logics behind OWL-DL and OWL-Lite, are not AGM-compliant. Some works [6, 7] present rational sets of postulates that could replace the AGM paradigm, but they use belief sets too.

Although less intuitive, belief base revision [3] became a good alternative because it relies on finite sets and moreover, it can be used with any compact and monotonic logic as showed in [8]. In particular it can be used with any description logic that receives just finite inputs. Although this works in theory, the constructions for revision rely on the existence of negation in the logic and many description logics do not admit the negation of all kinds of axioms. In this paper, we propose two new options for belief base revision operations and discuss the relation between them and the ones in the literature.

The paper proceeds as follows: in Section 2 we introduce the concepts of belief revision that we will use and describe briefly some previous attempts to apply belief revision to description logics. In Section 3, we present two new operations characterized by sets of postulates and constructions. Then in Section 4, we compare our operations to two other existing operations and show the relation between them. Finally, in Section 5 we conclude and point towards future work.

2 Belief Revision and Description Logics

In this section, we briefly introduce the area of Belief Revision and some previous proposals to apply it to Description Logics. We first introduce the most widely used theory for belief revision, known as the AGM paradigm due to the authors of the seminal paper [4]. Then we discuss two proposals of applying the AGM paradigm to description logics and their shortcomings. We next show an alternative to AGM theory that is more suitable for computational needs and extensively studied in the literature.

2.1 AGM theory and Description Logics

Traditionally, in AGM theory [4, 2, 9], the beliefs of an agent are represented by a set of formulas closed under logical consequence, the *belief set*. Although the logic used is not specified, there are several assumptions made which hold for classical logic.

Three operations can be performed on belief sets: contraction, expansion and revision. Contraction consists in giving up as many beliefs as needed so that the new belief set does not imply a specified sentence. Expansion consists in adding information to the belief set. The result of an expansion can be a inconsistent set. Revision is consistent incorporation of new information, i.e., if the input sentence is consistent then the new belief set will be consistent (even if the old belief set was not). If necessary, consistence is obtained by deleting some parts of the original belief set. The operations are characterized by a set of axioms (the rationality postulates) and several constructions have been proposed together with representation theorems with respect to the postulates.

There are two main problems when one wishes to apply the original AGM theory to practical problems. The first is the fact that the belief sets are assumed to be closed under a consequence operator Cn, i.e., if K is a belief set, then $K = Cn(K)$. This usually means dealing with infinite sets. The second problem is that the consequence operator Cn is assumed to be tarskian, compact, satisfy the deduction theorem and supraclassicality. Following [10], we will refer to these properties as the AGM-assumptions. The AGM-assumptions exclude many interesting logics, such as many description logics.

Flouris et al. [10] have defined a class of *AGM-compliant logics*, that is, logics in which a contraction operator can be defined satisfying the AGM postulates. They have shown that all logics satisfying the AGM-assumptions (but not only them) are AGM-compliant. As a negative result, he has shown that some important description logics, such as SHOIN and SHIFF are not AGM-compliant.

Later works [7, 6] presented sets of postulates that could replace the AGM postulates for contraction and be used with a wider class of logics. Using the set of postulates proposed in [6], any tarskian and compact logic (in particular SHOIN and SHIFF) allows for a contraction operator satisfying the postulates.

However, if we want to use belief revision in real applications we should not use infinite belief sets. For this reason, in the rest of this paper, we concentrate on belief revision operations on belief bases, i.e., sets of formulas not necessarily closed under the consequence operator.

Flouris [5] has studied the applicability of base operations to description logics. He follows the work by Fuhrmann [11], where the AGM postulates have been adapted to deal with bases. It was already noted in the literature that these postulates were not suitable for belief base operations, due to the recovery postulate, but Flouris has shown the properties needed in order for a logic to be *base-AGM-compliant*. The conditions are stronger than the ones for the belief set case, which exclude the logics we are interested in. This is why we chose to follow Hansson's approach, where new sets of postulates were designed specific for the belief base case.

2.2 Belief Base Operations

Belief bases have been introduced in the literature [12, 13] as an alternative for representing the beliefs of an agents. Belief bases are (usually finite) sets of formulas not necessarily closed under logical consequence. The three operations for belief change in the AGM paradigm can be adapted for belief bases. We follow here the formalization in [3].

Of the three AGM operations only expansion is characterized in an unique way. When a belief base B is expanded with a proposition α, the resulting set $B + \alpha$ is obtained by simply adding the new belief to the old belief set:

$B + \alpha = B \cup \{\alpha\}$

Contraction and revision for belief bases are not uniquely defined, but as in AGM theory, constrained by sets of postulates. Unlike AGM theory, the different constructions proposed for contraction of belief bases are not equivalent (similarly for revision), i.e., there is not a single set of postulates that characterizes all constructions.

We will present in this section the operation of kernel contraction proposed by Hansson in [14] and the set of postulates that characterizes this construction.

The construction makes use of the concept of kernel, the set of minimal subsets of a given set that imply a given sentence:

Definition 1 *Let B be a set of formulas and α a formula. The kernel $B \perp\!\!\!\perp \alpha$ of B and α is defined as follows. For any set Y, $Y \in B \perp\!\!\!\perp \alpha$ if and only if:*

- $Y \subseteq B$
- $\alpha \in Cn(Y)$
- *For all Y' such that $Y' \subset Y$, $\alpha \notin Cn(Y')$*

An incision function σ selects at least one element of each kernel set. The idea is that σ picks up enough elements from the kernel so that if these elements were taken out of the belief base, then the new set would not imply the given sentence.

Definition 2 *An incision function σ for B is a function that for all α:*

- $\sigma(B \perp\!\!\!\perp \alpha) \subseteq \bigcup(B \perp\!\!\!\perp \alpha)$
- *If $\emptyset \neq X \in B \perp\!\!\!\perp \alpha$, then $X \cap \sigma(B \perp\!\!\!\perp \alpha) \neq \emptyset$*

Now we can define a kernel contraction for B:

Definition 3 *Let B be a belief base, α a formula and σ an incision function for B. The kernel contraction of B by α is defined as:*
$B -_\sigma \alpha = B \setminus \sigma(B \perp\!\!\!\perp \alpha)$

Hansson has shown that:

Theorem 1 *[14] The operator $-$ for B is a kernel contraction if and only if it satisfies the following postulates:*

[**success**] *If $\alpha \notin Cn(\emptyset)$, then $\alpha \notin Cn(B - \alpha)$.*
[**inclusion**] $B - \alpha \subseteq B$.
[**core-retainment**] *If $\beta \in B$ and $\beta \notin B - \alpha$, then there is a set B' such that $B' \subseteq B$ and $\alpha \notin Cn(B')$, but $\alpha \in Cn(B' \cup \{\beta\})$.*
[**uniformity**] *If it holds that for all subsets B' of B that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$.*

The first postulate says that the result of a contraction should not imply the contracted sentence. Inclusion says that the new belief base should not contain anything that was not already in the original set. The postulate of core-retainment tries to capture the intuition that if a sentence has to be removed, then this sentence is relevant to imply α. The last postulate says that if every subset of B that implies α implies β then contracting α should be the same as contracting β and vice-versa.

The representation theorem for this construction was shown in [8] to hold for any compact and monotonic logic. So it holds for a large variety of description logics, in particular for SHOIN and SHIFF.

In AGM theory and in most works deriving from it, revision operators (*) are related to contraction operators (-) via the *Levi identity*:

$B * \alpha = (B - \neg\alpha) + \alpha$

Hansson [15] has proposed an operation of *external revision* using the *reversed Levi identity*:

$B * \alpha = (B + \alpha) - \neg\alpha$

There are several possibilities for constructing an external revision operator. We will present here one based on kernel contraction.

Definition 4 *Let - be a kernel contraction operator. The operator of external kernel revision is defined by:*

$$B * \alpha = (B + \alpha) - \neg\alpha$$

Hansson has shown that:

Theorem 2 *[8] The operator $*$ for B is an external kernel revision if and only if it satisfies the following postulates:*

- [**non-contradiction**] *If $\neg\alpha \notin Cn(\emptyset)$ then $\neg\alpha \notin Cn(B * \alpha)$*
- [**inclusion**] $B * \alpha \subseteq B + \alpha$
- [**core-retainment**] *If $\beta \in B$ and $\beta \notin B * \alpha$, then there is some B' such that $B' \subseteq B \cup \{\alpha\}$ and $\neg\alpha \notin Cn(B')$, but $\neg\alpha \in Cn(B' \cup \{\beta\})$*
- [**success**] $\alpha \in B * \alpha$
- [**weak-uniformity**] *If $\alpha, \beta \in B$ and for all $B' \subseteq B$ we have that $\neg\alpha \in Cn(B')$ if and only if $\neg\beta \in Cn(B')$ then $B \cap (B * \alpha) = B \cap (B * \beta)$.*
- [**pre-expansion**] $B + \alpha * \alpha = B * \alpha$

In [8] it was shown that the theorem above holds for any monotonic and compact logic satisfying a property called α-local non-contravention:

Definition 5 *A consequence operator Cn satisfies α-local non-contravention if and only if, if $\neg\alpha \in Cn(B \cup \{\alpha\})$, then $\neg\alpha \in Cn(B)$.*

This characterization of revision, as well as the construction, relies on the existence of negation in the logic. This is not the case for several description logics. In particular, SHIFF and SHOIN do not admit negation of all kinds of axioms.

We can re-write the postulates of external kernel revision in a way such they do not use negation:

[weak consistency] If $\bot \notin Cn(\alpha)$ then $\bot \notin Cn(B * \alpha)$
[inclusion] $B * \alpha \subseteq B + \alpha$
[core-retainment] If $\beta \in B$ and $\beta \notin B * \alpha$, then there is some B' such that $B' \subseteq B$ and $\bot \notin Cn(B' \cup \{\alpha\})$, but $\bot \in Cn(B' \cup \{\beta, \alpha\})$
[success] $\alpha \in B * \alpha$
[weak uniformity] If $\alpha, \beta \in B$ and for all $B' \subseteq B$ we have that $\bot \in Cn(B' \cup \{\alpha\})$ if and only if $\bot \in Cn(B' \cup \{\beta\})$ then $B \cap (B * \alpha) = B \cap (B * \beta)$.
[pre-expansion] $B + \alpha * \alpha = B * \alpha$

It is not difficult to see that in logics where negation has a classical behavior (i.e., satisfies explosiveness[1] and α-local-non-contravention), this set of postulates is equivalent to the previous one.

Another idea would be to define the negation of an axiom in an abstract way and then try to find the negation of every axiom that could be built. This approach was followed in [16], but for belief sets. The authors define consistency negation of an axiom in a generic way. Their goal was to find a definition of negation that: exists in (almost) every DL, the definition coincides with classical negation if applied to classical logic and checking if an axiom is the negation of another should be decidable. We could try to adapt this approach for belief bases, but this will be left for a future work. In this work we will follow a different approach.

In the next section, we propose a new construction for revision, together with an axiomatic characterization, that does not depend directly on the notion of negation.

3 Revision without negation

In the last Section, we have seen constructions for belief base operators of contraction and revision which are characterized by sets of postulates. We have seen that the constructions and postulates can be used with a large class of logics. Usually revision is built using contraction and the Levi identity, or its reverse. This is not easy when we are dealing with description logics. In order to use the Levi identity we have to add an axiom and then contract by the negation of it. The difficulty arises when we try to find the negation of an axiom. In many description logics the negation of some axioms is not defined. This problem was already mentioned in [10] and [5].

Our construction is based on an alternative operation proposed by Hansson, *semi-revision* [17]. A usual operator of revision guarantees, through the success postulate, that after the revision a given formula is added to the belief base. An operator of semi-revision does not satisfy the success postulate. So the result of a semi-revision is always a consistent set such that the formula by which we have revised does not necessarily belong to it.

[1] A consequence operator Cn satisfies explosiveness if and only if, for all α and β, $\beta \in Cn(\{\alpha, \neg\alpha\})$.

First a particular construction for an operator of semi-revision will be presented with the associated set of postulates and representation theorem. Then this operator will be adapted to an operator of belief base revision, i.e., an operator that satisfies the success postulate.

The semi-revision operator is built by adding the formula and then contracting the inconsistency:

Definition 6 *[17] Let B be a belief base, α a formula and $-$ a kernel contraction for B. The kernel semi-revision of B by α is defined as:*
$B?\alpha = (B + \alpha) - \bot$

Theorem 3 *[17]*
The operator ? for B is a kernel semi-revision if and only if it satisfies the following postulates:

[**consistency**] $\bot \notin Cn(B?\alpha)$
[**inclusion**] $B?\alpha \subseteq B + \alpha$
[**core-retainment**] *If $\beta \in B$ and $\beta \notin B?\alpha$ then there is at least one B' such that $B' \subseteq B + \alpha$ and $\bot \notin Cn(B')$, but $\bot \in Cn(B' \cup \{\beta\})$*
[**pre-expansion**] $(B + \alpha)?\alpha = B?\alpha$
[**internal exchange**] *If $\alpha, \beta \in B$ then $B?\alpha = B?\beta$*

It has been shown in [8] that the representation theorem above holds for any compact and monotonic logic such that $\bot \notin Cn(\emptyset)$.

In an operation of semi-revision, if the new formula is involved in an inconsistency, it may be given up when contracting by $\bot$. In order to transform it in a revision (satisfying the success postulate), we need to find a way to "protect" the new formula and make sure it stays in the revised belief base.

We present two different constructions for revision without negation, both based on semi-revision, with different properties. The first one assures that the revised base is always consistent, but success does not hold in case the formula to be added is inconsistent. In the second construction, success always holds, but if the formula to be added is inconsistent, the resulting revised base is inconsistent. This is more in line with the AGM paradigm.

3.1 Weak success

For the first construction, we have to restrict the incision functions that can be used:

Definition 7 *An incision function that protects consistent inputs is a function $\overline{\sigma}$ such that:*

- $\overline{\sigma}(\alpha, B \perp\!\!\!\perp \bot) \subseteq \bigcup(B \perp\!\!\!\perp \bot)$
- *If $\emptyset \neq X \in B \perp\!\!\!\perp \bot$, then $X \cap \overline{\sigma}(\alpha, B \perp\!\!\!\perp \bot) \neq \emptyset$*
- *If $\bot \notin Cn(\{\alpha\})$, then $\alpha \notin \overline{\sigma}(\alpha, B \perp\!\!\!\perp \bot)$*

The idea is that the incision function preserves α whenever it is possible, i.e., whenever α is not a contradiction by itself. If α is consistent, then there is no set in $B \perp\!\!\!\perp \bot$ containing only α, so it is always possible to choose other elements for the incision.

We can now define a semi-revision that protects consistent inputs:

Definition 8 *A kernel semi-revision with weak success is defined as:*
$B?_{\overline{\sigma}}\alpha = (B + \alpha) \setminus \overline{\sigma}(\alpha, B + \alpha \perp\!\!\!\perp \bot)$

A semi-revision protecting the input adds a formula and then removes all the inconsistencies, like the usual semi-revision, but it never chooses α to be removed, unless α is inconsistent.

Theorem 4 *Let Cn be a compact and monotonic consequence operator. The operator ? for B is a kernel semi-revision with weak success if and only if it satisfies the following postulates:*

[**weak success**] *If $\bot \notin Cn(\{\alpha\})$, then $\alpha \in B?\alpha$*
[**consistency**] $\bot \notin Cn(B?\alpha)$
[**inclusion**] $B?\alpha \subseteq B + \alpha$
[**core-retainment**] *If $\beta \in B$ and $\beta \notin B?\alpha$ then there is at least one B' such that $B' \subseteq B + \alpha$ and $\bot \notin Cn(B')$, but $\bot \in Cn(B' \cup \{\beta\})$*
[**pre-expansion**] $(B + \alpha)?\alpha = B?\alpha$

Proof: (i) construction $\Rightarrow$ postulates:

Let $?_{\overline{\sigma}}$ be an operator of kernel semi-revision with weak success based on an incision function that almost protects the input, $\overline{\sigma}$. It follows directly from the construction that *inclusion* and *pre-expansion* are satisfied. From the definition of an incision function that almost protects the input, it follows that $?_{\overline{\sigma}}$ satisfies *weak success* and *consistency.* Finally, for *core-retainment*, let $\beta \in B \setminus B?_{\overline{\sigma}}\alpha$. Then by construction $\beta \in \overline{\sigma}(\alpha, (B \cup \{\alpha\}) \perp\!\!\!\perp \bot)$. This means that for some set $X \in (B \cup \{\alpha\}) \perp\!\!\!\perp \bot$, $\beta \in X$. Let $B' = X \setminus \{\beta\}$. We have $B' \subseteq B \cup \{\alpha\}$, $\bot \notin Cn(B')$ and $\bot \in Cn(B' \cup \{\beta\})$.

(ii) postulates $\Rightarrow$ construction: Let ? be an operator satisfying the postulates above and let σ be such that for every formula α:
$\sigma(\alpha, B \perp\!\!\!\perp \bot) = B \setminus (B?\alpha)$

We have to show (1) that σ is an incision function that almost protects the input for the given domain and (2) that $B?\alpha = B?_{\sigma}\alpha$.

(1) We have to show that the three conditions of Definition 7 are satisfied. For the first condition, let $\beta \in \sigma(\alpha, B \perp\!\!\!\perp \bot)$. Then it holds that $\beta \in B \setminus (B?\alpha)$ and it follows from *core-retainment* that there is some $B' \subseteq B + \alpha$ such that $\bot \notin Cn(B')$ and $\bot \in Cn(B' \cup \{\beta\})$. It follows that there is some subset B'' of B' such that $B'' \cup \{\beta\} \in B \perp\!\!\!\perp \bot$ and hence, $\beta \in \bigcup(B \perp\!\!\!\perp \bot)$.

For the second condition, let $\emptyset \neq X \in B \perp\!\!\!\perp \bot$. Suppose that $X \cap \sigma(\alpha, B \perp\!\!\!\perp \bot) = \emptyset$. Then $X \subseteq B?\alpha$. Since $\bot \in Cn(X)$, it follows from monotony that $\bot \in Cn(B?\alpha)$, contrary to *consistency.* This contradiction is sufficient to prove that $X \cap \sigma(\alpha, B \perp\!\!\!\perp \bot) \neq \emptyset$.

For the third condition, suppose $\bot \notin Cn(\{\alpha\})$. By *weak success*, $\alpha \in B?\alpha$, and hence, $\alpha \notin \sigma(\alpha, B \perp\!\!\!\perp \bot)$.

(2) By definition, $\sigma(\alpha, (B \cup \{\alpha\}) \perp\!\!\!\perp \bot) = (B \cup \{\alpha\}) \setminus ((B \cup \{\alpha\})?\alpha) = (B \cup \{\alpha\}) \setminus B?\alpha$ (*pre-expansion*). Hence, $B?_\sigma\alpha = (B \cup \{\alpha\}) \setminus \sigma((B \cup \{\alpha\}) \perp\!\!\!\perp \bot) = (B \cup \{\alpha\}) \setminus ((B \cup \{\alpha\}) \setminus B?\alpha) = B?\alpha$ (*inclusion*). □

3.2 Success

We now define an operation of belief base revision that does not rely on negation and has the property of success.

Definition 9 *An incision function that protects the input is a function σ such that:*

- $\sigma(\alpha, B \perp\!\!\!\perp \bot) \subseteq \bigcup(B \perp\!\!\!\perp \bot)$
- *If $\bot \notin Cn(\{\alpha\})$ and $\emptyset \neq X \in B \perp\!\!\!\perp \alpha$, then $X \cap \sigma(\alpha, B \perp\!\!\!\perp \bot) \neq \emptyset$*
- $\alpha \notin \sigma(\alpha, B \perp\!\!\!\perp \bot)$

The idea here is that the incision function always preserves α. In the case where α is a contradiction, not enough will be chosen to make the resulting belief base consistent. In particular, when α is inconsistent we could have an incision function that protects α that returns the empty set and in this case, $B * \alpha = B + \alpha$.

Definition 10 *A kernel revision without negation is defined as:*
$B *_\sigma \alpha = (B + \alpha) \setminus \sigma(\alpha, B + \alpha \perp\!\!\!\perp \bot)$

Theorem 5 *Let Cn be a compact and monotonic consequence operator. The operator $*$ for B is a kernel revision without negation if and only if it satisfies the following postulates:*

[**success**] $\alpha \in B * \alpha$
[**weak consistency**] *If $\bot \notin Cn(\{\alpha\})$, then $\bot \notin Cn(B * \alpha)$*
[**inclusion**] $B * \alpha \subseteq B + \alpha$
[**core-retainment**] *If $\beta \in B$ and $\beta \notin B * \alpha$ then there is at least one B' such that $B' \subseteq B + \alpha$ and $\bot \notin Cn(B')$, but $\bot \in Cn(B' \cup \{\beta\})$*
[**pre-expansion**] $(B + \alpha) * \alpha = B * \alpha$

Proof: (i) construction ⇒ postulates:

Let $*_\sigma$ be an operator of kernel revision without negation based on an incision function σ that protects the input. It follows directly from the construction that *inclusion* and *pre-expansion* are satisfied. From the definition of an incision function that protects the input it follows that $*_\sigma$ satisfies *success* and *weak consistency*. Finally, for *core-retainment*, let $\beta \in B \setminus B *_\sigma \alpha$. Then by construction $\beta \in \sigma(\alpha, (B \cup \{\alpha\}) \perp\!\!\!\perp \bot)$. This means that for some set $X \in (B \cup \{\alpha\}) \perp\!\!\!\perp \bot$, $\beta \in X$. Let $B' = X \setminus \{\beta\}$. We have $B' \subseteq B \cup \{\alpha\}$, $\bot \notin Cn(B')$ and $\bot \in Cn(B' \cup \{\beta\})$.

(ii) postulates $\Rightarrow$ construction: Let $*$ be an operator satisfying the postulates above and let σ be such that for every formula α:

$\sigma(\alpha, B \perp\!\!\!\perp \bot) = B \setminus (B * \alpha)$

We have to show (1) that σ is an incision function that protects the input for the given domain and (2) that $B * \alpha = B *_\sigma \alpha$.

(1) We have to show that the three conditions of Definition 9 are satisfied. For the first condition, let $\beta \in \sigma(\alpha, B \perp\!\!\!\perp \bot)$. Then it holds that $\beta \in B \setminus (B * \alpha)$ and it follows from *core-retainment* that there is some $B' \subseteq B + \alpha$ such that $\bot \notin Cn(B')$ and $\bot \in Cn(B' \cup \{\beta\})$. It follows that there is some subset B'' of B' such that $B'' \cup \{\beta\} \in B \perp\!\!\!\perp \bot$ and hence, $\beta \in \bigcup(B \perp\!\!\!\perp \bot)$.

For the second condition, let $\bot \notin Cn(\{\alpha\})$ and $\emptyset \neq X \in B \perp\!\!\!\perp \bot$. Suppose that $X \cap \sigma(\alpha, B \perp\!\!\!\perp \bot) = \emptyset$. Then $X \subseteq B * \alpha$. Since $\bot \in Cn(X)$, it follows from monotony that $\bot \in Cn(B * \alpha)$, contrary to *weak consistency.* This contradiction is sufficient to prove that $X \cap \sigma(\alpha, B \perp\!\!\!\perp \bot) \neq \emptyset$.

For the third condition, it suffices to note that by *success*, $\alpha \in B * \alpha$, and hence, $\alpha \notin \sigma(\alpha, B \perp\!\!\!\perp \bot)$.

(2) By definition, $\sigma(\alpha, (B \cup \{\alpha\}) \perp\!\!\!\perp \bot) = (B \cup \{\alpha\}) \setminus ((B \cup \{\alpha\}) * \alpha) = (B \cup \{\alpha\}) \setminus B * \alpha$ (*pre-expansion*). Hence, $B *_\sigma \alpha = (B \cup \{\alpha\}) \setminus \sigma((B \cup \{\alpha\}) \perp\!\!\!\perp \bot) = (B \cup \{\alpha\}) \setminus ((B \cup \{\alpha\}) \setminus B * \alpha) = B * \alpha$ (*inclusion*). □

4 Comparing the operations

We have defined two different operations that can be used to revise ontologies in description logics. In this section we compare them to other proposals according to their properties.

In [18], the authors proposed the use of semi-revision for revising ontologies. The problem with semi-revision is that there is no guaranty that the new formula will be in the revised ontology, or even implied by it. We can see the operations described in the previous section as a link between semi-revision and revision.

Starting from semi-revision, when we switch to the operation defined in Section 3.1, we get some guaranty of success, through the *weak success* postulate. Whenever the new formula is consistent, it will be incorporated to the belief base. The price we pay for weak success is the loss of *internal exchange*. This means that, even if α and β are elements of B, the result of revising by them may be different.

From the operation defined in Section 3.1 to the one defined in Section 3.2, we get the *success* postulate, but loose unconditional *consistency.* This means that the resulting revised base may end up being inconsistent, but this only happens if the formula being added is inconsistent. This operation is closer to AGM-style intuitions. What is missing from Hansson's external revision is some form of uniformity. This is due to the syntactical nature of "protecting" α. One could think about other definitions of protection that would take into account logically equivalent formulas, or set of formulas, but this is left for future work.

In the rest of this section, we present a short example to illustrate the use of the revision operations. The example is described in a simple description logic.

Let us consider the following knowledge base B about Tweety:

$$Bird \sqsubseteq Fly \tag{1}$$

$$Bird(Tw) \tag{2}$$

$$Peng \sqsubseteq \neg Fly \tag{3}$$

$$\neg Peng(Tw) \tag{4}$$

The base contains thus information about birds (that they fly), penguins (that they do not fly) and an individual, Tweety, that is a bird and not a penguin.

Now assume that we receive information that we were wrong and that Tweety is a penguin, i.e., we want to add a new formula α to B:

$\alpha = Peng(Tw)$

4.1 Semi-Revision

In order to find the resulting knowledge base $B?\alpha$, we first have to compute the kernel set $B \cup \{\alpha\} \perp\!\!\!\perp \bot$.

$B \cup \{\alpha\} \perp\!\!\!\perp \bot = \{\{\neg Peng(Tw), Peng(Tw)\},$
$\{Bird \sqsubseteq Fly, Bird(Tw), Peng \sqsubseteq \neg Fly, Peng(Tw)\}\}$

There are several different possibilities for an incision function, we could have, for example:

$\sigma(B \cup \{\alpha\} \perp\!\!\!\perp \bot) = \{Peng(Tw)\}$

This would mean that the new information is not accepted and the resulting base is equal to the original one:

$B?\alpha = B + \alpha \setminus \{\alpha\}$

4.2 Semi-revision with weak success

In this case, we need an incision function that almost protects α, i.e., if α is consistent, it is never chosen. Since $Peng(Tw)$ is consistent, the incision function has to choose at least one element different from α of each set in $B \cup \{\alpha\} \perp\!\!\!\perp \bot$. One possible choice is:

$\overline{\sigma}(B \cup \{\alpha\} \perp\!\!\!\perp \bot) = \{\neg Peng(Tw), Bird \sqsubseteq Fly\}$

For this example the resulting set is:

$K?_{\overline{\sigma}}\alpha = \{Bird(Tw), Peng \sqsubseteq \neg Fly, Peng(Tw)\}$

4.3 Revision without negation

This operation only differs from semi-revision with weak success in the case where the new formula is inconsistent. Since in this example, the formula being added is consistent, an incision function that almost protects α is also an incision function that protects α and the result may be the same with the two operations.

4.4 External kernel revision

For external kernel revision, we have to compute the kernel set $B \cup \{\alpha\} \perp\!\!\!\perp \neg\alpha$:

$$B \cup \{\alpha\} \perp\!\!\!\perp \neg\alpha = \{\{\neg Peng(Tw)\}, \{Bird \sqsubseteq Fly, Bird(Tw), Peng \sqsubseteq \neg Fly\}\}$$

We could obtain the same result as in the two previous operations, by taking an incision function such that:

$$\sigma(B \cup \{\alpha\} \perp\!\!\!\perp \neg\alpha) = \{\neg Peng(Tw), Bird \sqsubseteq Fly\}$$

Note however, that this operation and revision without negation are not equivalent, the postulate of core-retainment is not identical in both cases. In fact we can show that if an operation satisfies the core-retainment of external kernel revision then it satisfies the core-retainment of revision without negation, but this is not true in the other way. Intuitively in the second any formula that is responsible for the deriving the inconsistency can be removed, but in the first one only the ones that are responsible for deriving $\neg\alpha$ can be removed.

Moreover, this operation depends on the existence of negation: if we want to revise by $Peng \sqsubseteq Bird$, for example, in several description logics we cannot form the negation of this axiom and the traditional operation of revision cannot be used.

5 Conclusions and Future Work

In [8], several operations of belief base change were generalized and shown to hold for a wide range of logics. The operations for contraction can be used with any compact and monotonic logic, and are thus applicable to ontologies represented in most description logics. Revision, on the other side, is usually defined based on contraction using negation. In several description logics, there is no negation for all kinds of axioms.

In this paper we have proposed two new operations of belief base revision, characterized by sets of postulates and constructions. The idea was to get rid of the dependence on negation.

In [16], the authors propose two different possible definitions for negation in ontologies. Future works include testing this notions of negation with the (reversed) Levi identity in order to see whether we get a meaningful revision operator.

We also plan to implement and test the ideas presented here. When trying to implement the operations defined in the previous sections, we are confronted with the problem of finding the kernel sets, which is the computational bottleneck of these constructions. Fortunately this problem has already been addressed for description logics. Description logic systems typically offer a set of inference services, such as: consistency checking, classification and concept subsumption. In [19] the authors proposed that the systems should provide debugging services. One of these debugging services was called axiom pinpointing. Axiom pinpointing is an inference service that given a sentence provides the set of all justifications for this sentence. By justification the author mean the smallest sets of formulas from the knowledge base (KB) that imply the sentence. In other words, axiom pinpointing returns the kernel of the KB given a sentence.

In [19] the authors show two ways of computing axiom pinpointing. One way was called black-box because it is reasoner-independent, the reasoner is used as an oracle that tells if a concept is satisfiable. The second way was called glass-box because it is reasoner-dependent. Though the first way is more general, the second way is much more efficient. The glass-box algorithm showed in [19] was based on the tableaux decision procedure for concept satisfiability in SHOIN.

Both services return only one justification for the sentence (one element of the kernel). To find all justifications, an algorithm based on Reiter's Hitting Set Tree Algorithm [20] could be used, which would also find the possible incision functions. The use of Reiter's algorithm for implementing belief change operators was suggested in [21] and used in [18].

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