On the Relation Between Remainder Sets and Kernels

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Abstract

There are two main constructions for contraction in belief bases: partial meet contraction (that depends on the remainder set of the base) and kernel contraction (that depends on the kernel of the base). It is well known that kernel contraction is more general than partial meet contraction.

What is shown in the paper is a way to derive the whole remainder set from the kernel without further calls to a theorem prover. This result shows that finding the kernel of a base is at least as difficult as finding the remainder set of the same base.

1 Introduction

Belief revision deals with the problem of accomodating new information in an existing body of beliefs. When dealing with finite belief bases, there are two main constructions that are in a sense dual of each other: partial meet and kernel constructions. Partial meet constructions were proposed by Alchourrn and Makinson [AM82] and depend on finding the maximal subsets of the belief base that do not imply a given formula (the remainder set of the base). Kernel constructions were proposed by Hansson [Han94] as a generalization of safe contraction [AM85] and rely on finding the minimal subsets of the belief base that imply a given formula (the kernel of the base).

Because kernel constructions deal with minimal sets, it has sometimes been implicitly assumed that from the computational point of view they were more efficient than partial meet, that depend on finding maximal sets. In this paper we show a way of deriving the remainder set of a base B and

a formula α from the kernel of B and α without calling a theorem prover. This result sugests that finding the remainder set of a base B is at least as easy as finding the kernel of B.

The paper is organized as follows: first we define partial meet contraction and remainder sets and present some properties of this type of contraction. Then kernel and kernel contraction are defined and we present some properties of this type of contraction too. The next section shows how to find the remainder set from the kernel without further calls to the theorem prover. Last we conclude and discuss future work.

2 Partial Meet Contraction

The idea of *partial meet contraction* is that if we want to give up a belief α from a belief base B, then we can look at the maximal subsets of B that do not imply α and take the intersection of some of them. The sets are selected by a *selection function*.

The remainder set of B and α ($B\perp\alpha$) is the set of maximal subsets of B that do not imply α. Formally:

Definition 2.1 (Remainder Set) $B' \in B\perp\alpha$ *iff* $B' \subseteq B$, $\alpha \notin Cn(B')$ *and* $B' \subset B''$ *implies that* $\alpha \in Cn(B'')$

Partial meet contraction ($B\dot{-}_\gamma\alpha$) is defined as the intersection of at least one element (selected by γ) of the remainder set of B and α.

Definition 2.2 (Partial Meet Contraction) $B\dot{-}_\gamma\alpha = \gamma\bigcap(B\perp\alpha)$ *where the selection function function γ must satisfy: (i)* $\emptyset \neq \gamma(B\perp\alpha) \subseteq B\perp\alpha$ *if* $B\perp\alpha \neq \emptyset$ *and (ii)* $\gamma(B\perp\alpha) = \{B\}$ *otherwise.*

The representation theorem for partial meet contraction shows which properties fully characterise the contraction:

Theorem 2.3 *[Han92] An operation $B\dot{-}\alpha$ is a partial meet contraction iff it satisfies:*

success: $\alpha \notin B\dot{-}\alpha$

inclusion: $B\dot{-}\alpha \subseteq B$

uniformity: *For all subsets B' of B it holds that $\alpha \in Cn(B')$ iff $\beta \in Cn(B')$ then $B\dot{-}\alpha = B\dot{-}\beta$.*

relevance: *If $\beta \in B$ and $\beta \notin B\dot{-}\alpha$, then there is B' such $B\dot{-}\alpha \subseteq B' \subseteq B$ and $\alpha \notin Cn(B')$, but $\alpha \in Cn(B' \cup \{\beta\})$.*

3   Kernel Contraction

The construction of *kernel contraction* consists in finding the minimal subsets of the belief base that imply the formula being contracted and then removing at least one element of each of these subsets.

The kernel of B and α is the minimal subset of B that implies α. Formally:

Definition 3.1 (Kernel) $B' \in B \perp\!\!\!\perp \alpha$ *iff* $B' \subseteq B$, $\alpha \in Cn(B')$ *and* $B'' \subset B'$ *implies that* $\alpha \notin Cn(B'')$

The kernel contraction $B \dot{-}_\sigma \alpha$ is defined choosing (with an *incision function* σ) at least one element of each kernel of B and α and taking it out of B.

Definition 3.2 (Kernel Contraction) $B \dot{-}_\sigma \alpha = B \setminus \sigma(B \perp\!\!\!\perp \alpha)$ *where the incision function* σ *must satisfy:* (i) $\sigma(B \perp\!\!\!\perp \alpha) \subseteq \bigcup(B \perp\!\!\!\perp \alpha)$ *and* (ii) *if* $\emptyset \neq X \in B \perp\!\!\!\perp \alpha$ *then* $X \cap \sigma(B \perp\!\!\!\perp \alpha) \neq \emptyset$

Kernel contraction does not satisfy the relevance postulate, but only a weaker postulate called core-retainment. In fact, the operation is characterized by success, inclusion, uniformity, and core-retainment:

Theorem 3.3 *[Han94] An operation $B \dot{-} \alpha$ is a kernel contraction iff it satisfies:*

success: $\alpha \notin B \dot{-} \alpha$

inclusion: $B \dot{-} \alpha \subseteq B$

uniformity: *For all subsets B' of B it holds that $\alpha \in Cn(B')$ iff $\beta \in Cn(B')$ then $B \dot{-} \alpha = B \dot{-} \beta$.*

core-retainment: *If $\beta \in B$ e $\beta \notin B \dot{-} \alpha$, then there is B' such $B' \subseteq B$ and $\alpha \notin B'$, but $\alpha \in Cn(B' \cup \{\beta\})$.*

It is easy to see that every operation that satisfies relevance also satisfies core-retainment. However, the converse is not true [Han97].

One interesting corollary of this property is that every partial meet contraction is a kernel contraction, but not the other way round.

4 Obtaining Remainders from the Kernel Set

What we are going to show in this section is that once we have the kernel of a set B, we can derive the remainder set of B without calling the theorem prover.

The main idea is that we can obtain each maximal subset of B that does not imply α as $B \setminus X$ where X is a minimal incision function for B and α.

Definition 4.1 (Minimal Incision Function) *$\sigma(B \perp\!\!\!\perp \alpha)$ is a minimal incision function iff there is no other incision function $\sigma'(B \perp\!\!\!\perp \alpha)$ such that $\sigma'(B \perp\!\!\!\perp \alpha) \subset \sigma(B \perp\!\!\!\perp \alpha)$*

Theorem 4.2 *$B \perp \alpha = \{B \setminus X | X$ is a minimal incision function$\}$*

The minimal incision functions of B can be obtained from the kernel of B using any algorithm for finding minimal hitting sets [Rei87, GSW89], as was shown in [Was00].

Although the complexity of the hitting set problem being NP-complete [FV04], it can be less than the complexity of the theorem prover, depending on which logic is being used. Furthermore, once the kernel is obtained the remainder set can be derived whitout any further calls to the theorem prover.

As far as we know there is no proof that one can obtain the remainder set from the kernel without making further calls to the theorem prover. The closest work in the literature in this direction is [FFKI06] that shows how to find a selection function from an incision function and vice-versa, but what we wanted is to find the whole kernel from the remainder set (like we can find the whole remainder set from the kernel). In fact we believe that this is not possible.

5 Conclusions and Future Work

The results presented here showed that the minimal numbers of calls to the theorem prover to find the remainder set of a set B is less or equal than the number of calls to the theorem prover to find the kernel of B. In other words, the problem of finding the kernel of B is at least as difficult as the problem of finding the remainder of B. This result suggests that finding the remainder set can be easier than finding the kernel but not the other way round. This suspect was confirmed by some preliminary experimental results.

Future work includes exploring ways to derive kernels from remainder sets and improving and further testing the implementation.

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